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An Indispensable Equation in the Deduction of E = mc^2

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Abstract

Einstein has given equation of relativistic variation of light energy in the paper widely known as Special Theory of Relativity. But this equation has two main limitations e.g. it does not obey principle of dimensional homogeneity and simple algebraic identity. Then this equation (relativistic variation of light energy) was used by Einstein in calculation of mass-light energy equation ($L= \Delta mc^2$) from which Einstein speculated $E=\Delta mc^2$ (holds good in all cases). The reason for these limitations is that in some cases both denominator and numerator become zero simultaneously.

1.0 Introduction

Einstein's June 1905 paper, titled 'On the Electrodynamics of Moving Bodies' is known as Special Theory of Relativity. This paper has two parts i.e. Kinematical Part and Electrodynamical Part. In the second part subsection 7 is 'Theory of Doppler's Principle and of Aberration' where equation for variation of light energy (l^*) with velocity (v) is introduced i.e.

$$l^* = l(1-\cos\phi\ v/c)/(1-v^2/c^2)^{1/2} \qquad (1)$$

Einstein called this equation as Doppler Effect for any velocities whatever.
Einstein applied algebraic identity $[a^2-b^2 = (a+b)(a-b)]$ to eq.(1) and explained it in the following way,

" When $\phi = 0$ the equation assumes the perspicuous form

$$l^* = l[(1-v/c)/1+v/c)]^{1/2} \qquad (2)$$

We see that, in contrast with the customary view, when $v = -c$ and $l^* = \infty$.".

Einstein did not explain what is meant by
v (velocity of system) = –c (speed of light),
and in this case infinite amount of energy is emitted.

In relativistic variation of mass, the mass becomes infinite when v = c. Also in relativistic length contraction; length reduces to zero if v = c. As far as eq.(1) is concerned, under this condition i.e. when v = c, the light energy becomes 0/0 (undefined). Where as in eq.(2) which is solved form of eq.(1) light energy comes 0. So entirely different and inconsistent results are produced.

The results of relativistic equation are significant under relativistic conditions i.e. when velocity (v) is comparable with that of light(c). But in this equation under relativistic conditions (v is comparable with c or $v \approx c$ or v = c) gives inconsistent results. This aspect needs to be carefully studied, as it is basis of mass energy inter-conversion, which is basic equation.

Table I Variation of l^* unsolved (Einstein's original equation), and solved equation by Einstein with help of algebraic identity, $a^2-b^2 = (a+b)(a-b)$.

Sr.No	v	$l^* = l(1-v/c)/(1-v^2/c^2)^{1/2}$	$l^* = l[(1-v/c)/1+v/c)]^{1/2}$
1	0	$l^* = l$	$l^* = l$
2	c/2	0.5773 l	0.5773 l
3	0.9c	0.2294 l	0.2294 l
4	0.9999c	0.007071 l	0.007071 l
5	0.999999c	0.000707 l	0.000707 l

6	$v \rightarrow c$	$l^* \rightarrow 0/0$	$l^* \rightarrow 0$
7	$v = c$	$l^* = 0/0$	$l^* = 0$
8	$v = -c$	$l^* = \infty$	$l^* = \infty$

2.0 Critical analysis of eq.(1)

Einstein used eq.(1) under classical conditions (applying binomial theorem) derived light energy –mass inter-conversion equation

$$L = \Delta mc^2 \qquad (3)$$

where L is light energy emitted (more specifically ΔL) when mass (Δm) decreases. Then without any derivation or logical statement Einstein, hypothesized that equation (3) can be written for every energy E (simply replacing L by E) i.e.

$$E = \Delta mc^2 \qquad (4)$$

It should be noted that eq.(4) is hypothesized from eq.(3), which is based upon eq.(1).

Einstein simply replaced L by E in eq.(3) to get eq.(4). Here E in eq.(4) stands for all possible energies of the universe (discovered or undiscovered) e.g.

(i) sound energy, (ii) heat energy, (iii) chemical energy,
(iv) nuclear energy, (v) magnetic energy, (vi) electrical energy,
(vii) energy emitted in form of invisible radiations,
(viii) energy emitted in cosmological and astrophysical phenomena
(xi) energy emitted volcanic reactions
(x) energies co-existing in various forms etc. etc.

The eq.(1) i.e. $L = \Delta mc^2$ is meant for light energy and not for all energies of the universe. Thus equation $E = \Delta mc^2$ is speculated NOT derived by Einstein.The critical analysis of eq.(1-3) is absolutely necessary as $E = \Delta mc^2$ holds good universally.

(i) Inconsistency with dimensional homogeneity

Einstein's above equation is meant for relativistic region i.e. when velocity is comparable to c. But under this conditions eq.(1) gives inconsistent results as discussed below.

But under this condition (when v = c or $v \rightarrow c$) the **unsolved equation i.e.**

$$l^* = l(1 - \cos\phi\, v/c)/(1 - v^2/c^2)^{1/2}$$

becomes

$$l^* = 0/0 \quad \text{or} \quad l^* \rightarrow 0/0$$

It contradicts the principle of dimensional homogeneity . The dimensions of LHS are that

of energy i.e. ML^2T^{-2} and that of RHS are undefined (physical quantity cease to exist).
The dimensions of LHS (l^*): dimensions of energy
Dimension of RHS (0/0): undefined
This is not justified by any law; hence it is inconsistency of eq.(1).

(ii) Inconsistency with basic algebraic identity $a^2–b^2 = (a+b)(a-b)$

Firstly consider an identity i.e. $(1^2 - 4^2)$ which can be solved as -15.

(b) Now as with $a^2–b^2 = (a+b)(a-b)$ we get

$(1^2 - 4^2) = (1+4)(1-4) = -15$

Hence the results are consistent with algebraic identity. However in case of eq.(1) i.e.

$$l^* = l(1–\cos\phi\ v/c)/ (1–v^2/c^2)^{1/2} \qquad (1)$$

The eq(1) does not obey the above algebraic identity. Einstein also simplified the eq.(1)

$$l^* = l[(1–v/c)/1+v/c)]^{1/2} \qquad (2)$$

(i) When v=c or $v \to c$ then eq.(1) does not obey the above algebraic identity . For example, unsolved equation i.e. eq.(1) when (ϕ =0, v = c or $v \to c$) gives results;

$l^* = 0/0$ or $l^* \to 0/0$

(ii) Whereas the same equation i.e. eq.(2) with help of algebraic identity $a^2-b^2 = (a+b)(a-b)$, then it gives different results i.e.

$l^* = 0$.

Hence the solved and unsolved forms of eq.(1) gives different results.

3.0 Conclusions, reasons and significance

There are two limitations of eq.(1)

Firstly it is **not consistent** with principle of dimensional homogeneity when $v = c$ or $v \rightarrow c$. When v is comparable with c then all the relativistic equations give significant results. Secondly, all the equations in solved or unsolved forms obey simple algebraic identity, but eq.(1) does not.

It is due to reason, that in this case numerator and denominator both become zero simultaneously under the condition when $v = c$. This is the main reason of the limitation, and while proposing any equation, it is kept in mind that denominator does not become zero. But eq.(1) does not obey this simple fact and denominator becomes zero along with numerator. These are further significant in the sense that Einstein used this eq.(1) to derive light energy mass equation i.e.

$$L = \Delta mc^2 \qquad (3)$$

From equation (1), which has serious limitations, Einstein further speculated the equation $E=\Delta mc^2$, from $L =\Delta mc^2$, which is based upon eq.(1) which have inconsistencies. The limitations of eq.(1) has far reaching impact on $E=\Delta mc^2$ which is exceptionally-2 significant equation.

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